# A NOTE ON UPPER AND LOWER δ-PRECONTINUOUS FUZZY MULTIFUNCTIONS

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**Abstract :**This paper is a continuation of [2]. In this paper a net in a topological space is used as a tool for characterizations of fuzzy upper (lower) δ-precontinuous multifunctions. Also we have established a mutual relationship between fuzzy upper (lower) semi-continuous multifunctions and fuzzy upper (lower) δ-precontinuous multifunctions.

**Keywords :** Fuzzy upper (lower) semi-continuity, fuzzy upper (lower) δ-precontinuity, $\delta_p$-convergence of a net in a topological space, limit point of a fuzzy net.

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**Introduction**

In 1985, Papageorgiou [7] introduced fuzzy multifunction, a function from an ordinary topological space $X$ to a fuzzy topological space $Y$ which carries an ordinary point of $X$ to a fuzzy set in $Y$ and from then a group of researchers are engaged themselves for studying different types of fuzzy multifunctions. Papageorgiou defined upper and lower inverses of a fuzzy multifunction. Afterwards, it was noticed in [6] that the definition of lower inverse of Papageorgiou was not natural and so this notion was redefined in [6] via q-coincidence and q-neighbourhoods of Pu and Liu [8] and ultimately some expected results were achieved in [6] by use of the new definition. With the definition of upper inverse as given in [7] and the definition of lower inverse as defined in [6], we introduce fuzzy upper (lower) δ-precontinuous multifunctions in [2] and in this paper we give some characterizations of this fuzzy multifunction specially via fuzzy nets.

Throughout this paper, $(X, \tau)$ or simply X will stand for an ordinary topological space, while by $(Y, \tau_Y)$ or simply by Y will always be denoted by a fuzzy topological space (fts, for short) in the sense of Chang [3]. The support of a fuzzy set A in Y will be denoted as supp A [11] and is defined by $\text{supp}\, A = \{y \in Y : A(y) \neq 0\}$. A fuzzy point [8] with the singleton support $y \in Y$ and the value $\alpha$ $(0 < \alpha \leq 1)$ at y will be denoted by $y_\alpha$. cl A and int A of a set A in X (respectively, a fuzzy set [11] in Y ) respectively stand for the closure and interior of A in X (respectively, in Y). $0_Y$ and $1_Y$ are the constant fuzzy sets taking respectively the constant values 0 and 1 on Y. The complement of a fuzzy set A in Y will be denoted by $1_Y \backslash A$ [11], defined by $(1_Y \backslash A)(y) = 1 - A(y)$, for each $y \in Y$. For two fuzzy sets A and B in Y, we write $A \leq B$ iff $A(y) \leq B(y)$, for each $y \in Y$, while we write A q B to mean A is quasi-coincident (q-coincident, for short) with B [8] if there is some $y \in Y$ such that $A(y) + B(y) > 1$; the negation of A q B is written as A $\bar{q}$ B. A (fuzzy) set A in X (resp. in Y) is called (fuzzy) regular open if $A = \text{int cl}\, A$( [1]). A fuzzy set A in Y is called a fuzzy neighbourhood (fuzzy nbd, for short) of a fuzzy set B [8] if there exists a fuzzy open set U in Y such that $B \leq U \leq A$. A fuzzy set B is called a quasi neighbourhood (q-nbd, for short) of a fuzzy set A in an fts Y if there is a fuzzy open set U in Y such that A q U $\leq$ B [8]. If, in addition, B is fuzzy open (regular open) then B is called a fuzzy open (regular open) q-nbd of A. In particular, a fuzzy set B in Y is a fuzzy open (resp. regular open) q-nbd of a fuzzy point $y_\alpha$ in Y if $y_\alpha$ q U $\leq$ B, for some fuzzy open (resp. regular open) set U in Y. The δ-interior [10] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by

$\delta - \text{int}\, A$. A subset A of X is called δ-open [10] if $A = \delta - \text{int}\, A$, i.e., a set is δ-open if it is the union of regular open sets. The complement of a δ-open set is called δ-closed. A set A of $(X, \tau)$ is δ-closed [10] iff $A = \delta\text{cl}\, A$ where $\delta\text{cl}\, A = \{x \in X : A \cap (\text{int cl}\, U) \neq \varphi, U \in \tau, x \in U\}$. A fuzzy point $x_\alpha$ is said to be a fuzzy δ-cluster point [4] of a fuzzy set A in an fts Y if every fuzzy regular open q-nbd $U$ of $x_\alpha$ is q-coincident with $A$. The union of all fuzzy δ-cluster points of $A$ is called the fuzzy δ-closure of $A$ and is denoted by $\delta cl\, A$ [4]. A subset $A$ is said to be δ-preopen [9] in $X$ if $A \subseteq int\,(\delta cl\, A)$. The family of all δ-preopen sets in $X$ is denoted by δ-PO(X) [9]. The δ-preinterior of a subset $A$ [9] of $X$ is defined to be the union of all δ-preopen sets contained in $A$ and is denoted by $\delta - int\, A$. The complement of a δ-preopen set is called δ-preclosed [9]. The intersection of all δ-preclosed sets containing $A$ in $X$ is called δ-preclosure of $A$ [9] and is denoted by $\delta - pcl\, A$. A set A is δ-preopen (δ-preclosed) iff $A = \delta - pint\, A$ (resp. $A = \delta - pcl\, A$) [9].

## 1. SOME WELL KNOWN DEFINITIONS AND THEOREMS

In this section we recall some definitions and theorems for ready references.

**DEFINITION 1.1.** [7] Let $(X, \tau)$ be an ordinary topological space and $(Y, \tau_1)$ be an fts. We say that $F : X \to Y$ is a fuzzy multifunction if corresponding to each $x \in X$, $F(x)$ is a unique fuzzy set in $Y$.

Henceforth by $F : X \to Y$ we shall mean a fuzzy multifunction in the above sense.

**DEFINITION 1.2.** [7,6] For a fuzzy multifunction $F : X \to Y$, the upper inverse $F^+$ and lower inverse $F^-$ are defined as follows :

For any fuzzy set $A$ in $Y$, $F^+(A) = \{x \in X : F(x) \leq A\}$, $F^-(A) = \{x \in X : F(x) q A\}$.

The relation between the upper and lower inverses of a fuzzy multifunction is an follows :

**THEOREM 1.3.** [6] For a fuzzy multifunction $F : X \to Y$, we have $F^{-}(1_Y \setminus A) = X \setminus F^{+}(A)$, for any fuzzy set $A$ in $Y$.

**DEFINITION 1.4.** [6] A fuzzy multifunction $F : X \to Y$ is said to be

(a) fuzzy upper semi-continuous at a point $x \in X$, if for each fuzzy open set $V$ in $Y$ with $F(x) \le V$, there exists an open set $U$ in $X$ containing $x$ such that $F(U) \le V$,

(b) fuzzy lower semi-continuous at a point $x \in X$, if for each fuzzy open set $V$ in $Y$ with $F(x)qV$, there exists an open set $U$ in $X$ containing $x$ such that $F(u)qV$, for all $u \in U$,

(c) Fuzzy upper (lower) semi-continuous if $F$ has this property at each point $x \in X$.

**DEFINITION 1.5.** [2] A fuzzy multifunction $F : X \to Y$ is said to be

(a) fuzzy upper δ-precontinuous at a point $x \in X$, if for each fuzzy open set $V$ in $Y$ with $F(x) \le V$, there exists $U \in \delta - PO(X)$ containing $x$ such that $F(U) \le V$,

(b) fuzzy lower δ-precontinuous at a point $x \in X$, if for each fuzzy open set $V$ in $Y$ with $F(x)qV$, there exists $U \in \delta - PO(X)$ containing $x$ such that $F(u)qV$, for all $u \in U$,

(c) Fuzzy upper (lower) δ-precontinuous if $F$ has this property at each point $x \in X$.

**THEOREM 1.6.** [2] A fuzzy multifunction $F : X \to Y$ is fuzzy upper δ-precontinuous iff $F^{+}(V) \in \delta - PO(X)$, for any fuzzy open set $V$ in $Y$.

**THEOREM 1.7.** [2] A fuzzy multifunction $F : X \to Y$ is fuzzy lower δ-precontinuous iff $F^{-}(V) \in \delta - PO(X)$, for any fuzzy open set $V$ in $Y$.

**REMARK 1.8.** It is clear from Definitions 1.4 and 1.5, that fuzzy upper (lower) semi-continuous multifunctions are fuzzy upper (lower) δ-precontinuous multifunctions. But the converse may not be true as seen from the following example.

**EXAMPLE 1.9.** Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{b\}, \{a,c\}\}$, $Y = [0,1]$, $\tau_Y = \{0_Y, 1_Y, B\}$ where $B(y) = 0.5$, for all $y \in Y$. Then $(X, \tau)$ and $(Y, \tau_Y)$ are topological space and an fts respectively.

Clearly, $\{b\},\{a,c\}$ are δ-open as well as δ-closed in $X$. Let $F : X \to Y$ be defined by $F(a) = A, F(b) = B, F(c) = C$ where $A(y) = 0.45, C(y) = 0.61$, for all $y \in Y$. $F^+(B) = \{x \in X : F(x) \le B\} = \{a,b\} (\subseteq \text{int}\,\delta cl(\{a,b\}) = \text{int}\,X = X) \in \delta - PO(X)$, $F^+(0_Y) = \phi, F^+(1_Y) = X$ and so by Theorem 1.6, $F^+$ is fuzzy upper δ-precontinuous multifunction. But $a \in X$, $F(a) = A \le B \in \tau_Y$. Then $\{a,c\}, X$ are the only open sets in $X$ containing $a$. $F(\{a,c\}) \not\le B$ as $F(c) = C \not\le B$. Then also $F(X) \not\le B$. Therefore, $F$ is not fuzzy semi-continuous at $a$. Again, $F^-(B) = \{x \in X : F(x) qB\} = \{c\}$ $(\subseteq \text{int}\,\delta cl(\{c\}) = \text{int}\{a,c\}$ $=$ $\{a,c\}) \in \delta - PO(X)$. Then by Theorem 1.7, $F$ is fuzzy lower δ-precontinuous. But $c \in X$, $F(c) = CqB \in \tau_Y$. The only open sets containing $c$ are $\{a,c\}$ and $X$. Now $F(\{a,c\})\bar{q}B$, for all $x \in \{a,c\}$ as $F(a) = A\bar{q}B$, $a \notin \{a,c\}$. Hence $F$ is not fuzzy lower semi-continuous at $c$.

Now we recall the δ-preregularity of a topological space from [9] under which fuzzy upper (lower) δ-precontinuity implies fuzzy upper (lower) semi- continuity.

**DEFINITION 1.10.** A topological space $(X,\tau)$ is said to be δ-preregular if for each δ-preclosed set $F$ in $X$ and each point $x \in X$ with $x \notin F$, there exists an open set $U$ and a δ-preopen set $V$ in $X$ such that $x \in U, F \subseteq V$ and $U \cap V = \emptyset$.

**THEOREM 1.11.** [9] A topological space $(X,\tau)$ is δ-preregular iff for each point $x \in X$ and each $U \in \delta - PO(X)$ containing $x$, there exists $V \in \tau$ such that $x \in V \subseteq \delta - pclV \subseteq U$.

**THEOREM 1.12.** Let $(X,\tau)$be a δ-preregular space and $F : X \to Y$ be fuzzy upper δ-precontinuous multifunction. Then $F$ is also fuzzy upper semi-continuous.

**PROOF.** Let $x \in X$ and $V$ be any fuzzy open set in $Y$ with $F(x) \le V$. As $F$ is fuzzy upper δ-precontinuous, there exists $U \in \delta - PO(X)$ containing $x$ such that $U \subseteq F^+(V)$. By Theorem 1.11, there exists an open set $W$ in $X$ such that $x \in W \subseteq \delta - pclW \subseteq U$. Then $x \in W \subseteq U \subseteq F^+(V)$. Hence $F$ is fuzzy upper semi-continuous.

**THEOREM 1.13.** Let $(X,\tau)$ be a δ-preregular space and $F:X\to Y$ be fuzzy lower δ-precontinuous multifunction. Then $F$ is also fuzzy lower semi-continuous.

**PROOF.** Let $x\in X$ and $V$ be any fuzzy open set in $Y$ with $F(x)qV$. As $F$ is fuzzy lower δ-precontinuous, there exists $U\in\delta-PO(X)$ containing $x$ such that $U\subseteq F^{-}(V)$. By Theorem 1.11, there exists an open set $W$ in $X$ such that $x\in W\subseteq\delta-pclW\subseteq U$. Then $x\in W\subseteq U\subseteq F^{-}(V)$. Hence $F$ is fuzzy lower semi-continuous.

## 2. CHARACTERIZATIONS OF FUZZY UPPER (LOWER) δ-PRECONTINUOUS MULTIFUNCTIONS VIA A NET (FUZZY NET) IN $X$ (IN $Y$).

In this section a new type of convergence of a net in an ordinary topological space has been introduced and we characterize fuzzy upper (lower) δ-precontinuous multifunctions via this newly defined convergence of a net. Also we define fuzzy inferior limit points of a fuzzy net and also a new type of convergence of a fuzzy net has been introduced and characterize fuzzy lower δ-precontinuous multifunctions via these new concepts.

**DEFINITION 2.1.** A net $\{S_n : n\in(D,\geq)\}$ in a topological space $X$ with the directed set $(D,\geq)$ as the domain, is said to $\delta_p$-converge to a point $x\in X$ if for each δ-preopen set $U\in X$ containing $x$, there exists $m\in D$ such that $S_n\in U$, for all $n\geq m$ $(n\in D)$.

**THEOREM 2.2.** For a fuzzy multifunction $F:X\to Y$, the following statements are equivalent :

(a) $F$ is fuzzy lower δ-precontinuous.

(b) For each fuzzy open set $V$ in $Y$, $F^{-}(V)\subseteq\delta-p\operatorname{int}F^{-}(V)$.

(c) For each fuzzy closed set $V$ in $Y$, $\delta-pclF^{+}(V)\subseteq F^{+}(V)$.

(d) For each point $x \in X$, if $\{S_n : n \in (D,\geq)\}$ is a net in $X$, $\delta_p$-converge to $x$, then for each fuzzy open set $V$ in $Y$ with $F(x)qV$, there exists $m \in D$ such that $F(S_n)qV$, for all $n \geq m$ ( $n \in D$).

**PROOF.** (a) $\Leftrightarrow$ (b) : Follows from Theorem 1.7.

(b) $\Rightarrow$ (c) : Let $G$ be a fuzzy closed set in $Y$. Then $1_Y \setminus G$ is fuzzy open in $Y$. By (b), $F^-(1_Y \setminus G) \subseteq \delta - p\operatorname{int} F^-(1_Y \setminus G) \Rightarrow X \setminus F^+(G) \subseteq X \setminus \delta - pclF^+(G) \Rightarrow \delta - pclF^+(G) \subseteq F^+(G)$.

(c) $\Rightarrow$ (b) : Retracing the above steps, we get the result.

(b) $\Rightarrow$ (d) : Let $\{S_n : n \in (D,\geq)\}$ be a net in $X$, $\delta_p$-converge to $x \in X$ and $V$ be a fuzzy open set in $Y$ with $F(x)qV$. Then $x \in F^-(V)$. By (b), there exists $U \in \delta - PO(X)$ containing $x$ such that $U \subseteq F^-(V)$. Since the net $\delta_p$-converge to $x$, and $U \in \delta - PO(X)$ containing $x$, by Definition 2.1, there exists $m \in D$ such that $S_n \in U$, for all $n \geq m$ $(n \in D) \Rightarrow S_n \in F^-(V)$, for all $n \geq m \Rightarrow F(S_n)qV$, for all $n \geq m$.

(d)$\Rightarrow$ (a) : If $F$ is not fuzzy lower δ-precontinuous at some point $x \in X$. Then there exists a fuzzy open set $V$ in $Y$ with $F(x)qV$ such that for each δ-preopen set $U$ in $X$ containing $x$ such that $F(x_U)\overline{q}V$, for some $x_U \in U$. Let $(D,\geq)$ be the directed set consisting of all pairs $(x_U, U)$ with $(x_U, U) \geq (x_W, W)$ iff $U \subseteq W$ ($U, W$ being any δ-preopen sets in $X$ containing $x$ and $F(x_U)\overline{q}U$, $F(x_W)\overline{q}W$) and consider the net $S(x_U, U) = x_U$ in $X$. Then evidently, the net $\{S_n : n \in (D,\geq)\}$ is $\delta_p$-convergent to $x$, but $F(S_n)\overline{q}V$, for each $n \in D$, contradicting (d).

**THEOREM 2.3.** For a fuzzy multifunction $F : X \to Y$, the following statements are equivalent :

(a) $F$ is fuzzy upper δ-precontinuous.

(b) For each fuzzy open set $V$ of $Y$, $F^{+}(V) \subseteq \delta - p\operatorname{int} F^{+}(V)$.

(c) For each fuzzy closed set $V$ of $Y$, $\delta - pclF^{-}(V) \subseteq F^{-}(V)$.

(d) For each point $x \in X$, if $\{S_n : n \in (D,\geq)\}$ is a net in $X$ which is $\delta_p$-convergent to $x$, then for each fuzzy open set $V$ in $Y$ with $x \in F^{+}(V)$, the net is eventually in $F^{+}(V)$.

**PROOF.** (a) $\Leftrightarrow$ (b) : Follows from Theorem 1.6.

(b) $\Rightarrow$ (c) : Let $G$ be a fuzzy closed set in $Y$. Then $1_Y \setminus G$ is fuzzy open in $Y$. By (b), $F^{+}(1_Y \setminus G) \subseteq \delta - p\operatorname{int} F^{+}(1_Y \setminus G) \Rightarrow X \setminus F^{-}(G) \subseteq X \setminus \delta - pclF^{-}(G) \Rightarrow \delta - pclF^{-}G) \subseteq F^{-}(G)$.

(c) $\Rightarrow$ (b) : Retracing the above steps, we get the result.

(b) $\Rightarrow$ (d) : Let $\{x_n : n \in (D,\geq)\}$ be a net in $X$, $\delta_p$-converge to $x$ and $V$ be a fuzzy open set in $Y$ with $x \in F^{+}(V)$. By (b), there exists $U \in \delta - PO(X)$ containing $x$ such that $U \subseteq F^{+}(V)$. Since the net $\delta_p$-converge to $x$, and $U \in \delta - PO(X)$ containing $x$, by Definition 2.1, there exists $m \in D$ such that $x_n \in U$, for all $n \geq m$ $(n \in D) \Rightarrow x_n \in F^{+}(V)$, for all $n \geq m$. Hence the net is eventually in $F^{+}(V)$.

(d)$\Rightarrow$ (a) : If $F$ is not fuzzy upper δ-precontinuous at some point $x \in X$. Then there exists a fuzzy open set $V$ in $Y$ with $F(x) \in V$ such that for each δ-preopen set $U$ in $X$ containing $x$ such that $F(x_U) \not\leq V$, for some $x_U \in U$. Let $(D,\geq)$ be the directed set consisting of all pairs $(x_U, U)$ with $(x_U, U) \geq (x_{V,} V)$ iff $U \subseteq V$ ($U,V$ being any δ-preopen sets in $X$ containing $x$) and consider the net $S(x_U, U) = x_U$ in $X$. Then evidently, the net $\{S_n : n \in (D,\geq)\}$ is $\delta_p$-convergent to $x$, but $F(S_n) \not\leq V$, i.e., $S_n \notin F^{+}(V)$ for each $n \in D$, contradicting (d).

**DEFINITION 2.4.** Let $(Y,\tau_1)$ be an fts and $\{S_n : n \in (D,\geq)\}$ be a fuzzy net in $Y$. A fuzzy point $y_\alpha$ in $Y$ is said to be a fuzzy inferior limit point of the net if for each fuzzy open q-nbd $V$ of $y_\alpha$, there exists $m \in D$ such that $S_n q V$, for all $n \geq m$ ($n \in D$). The union of all fuzzy inferior limit points of the net will be denoted by $f-\inf-LiS_n$.

**THEOREM 2.5.** A fuzzy multifunction $F: X \to Y$ is fuzzy lower δ-precontinuous at a point $x \in X$ iff for any net $\{S_n : n \in (D, \geq)\}$ in $X$, which $\delta_p$-converges to $x$, the net of fuzzy sets $\{F(S_n) : n \in (D, \geq)\}$ satisfies the relation $clF(x) \leq f-\inf-LiF(S_n)$.

**PROOF.** Let $F$ be fuzzy lower δ-precontinuous at $x \in X$ and $\{S_n : n \in (D, \geq)\}$ be a net in $X$, which $\delta_p$-converge to $x$. If $y_\alpha \leq clF(x)$, then for all fuzzy open q-nbd $V$ of $y_\alpha$, $VqF(x) \Rightarrow x \in F^-(V)$. Since $F$ is fuzzy lower δ-precontinuous, by Theorem 2.2 (a) ⇒ (b), $F^-(V) \subseteq \delta-p\text{int}\, F^-(V) \Rightarrow x \in U \subseteq F^-(V)$, for some $U \in \delta-PO(X)$ containing $x$. Then there exists $m \in D$ such that $S_n \in U \subseteq F^-(V)$, for all $n \geq m$ ($n \in D$), i.e., $F(S_n)\bar{q}V$, for all $n \geq m$. Consequently, $y_\alpha \in f-\inf-LiF(S_n)$.

Conversely, let $F$ be not fuzzy lower δ-precontinuous at $x$. Then by Theorem 2.2 (a) ⇔ (d), there exists a net $\{S_n : n \in (D, \geq)\}$ in $X$, which $\delta_p$-converges to $x$ and there exists a fuzzy open set $V$ in $Y$ with $F(x)qV$, such that for any $n \in D$, we have $F(S_m)\bar{q}V$, for some $m \geq n$ ($m \in D$). Now there exists $y \in \sup p(F(x) \cap V)$ such that $y_\alpha qV$ where $\alpha = [F(x)](y)$. Thus $y_\alpha \not\leq f-\inf-LiF(S_n)$. But $y_\alpha \leq F(x)$ and hence $y_\alpha \leq clF(x)$. Thus $clF(x) \not\leq f-\inf-LiF(S_n)$.

**DEFINITION 2.6.** Let $(Y, \tau_1)$ be an fts and $y_\alpha$ be a fuzzy point in $Y$. Then a fuzzy net $\{S_n : n \in (D, \geq)\}$ is said to converge to $y_\alpha$, written as $S_n \to y_\alpha$, if for any fuzzy open q-nbd $U$ of $y_\alpha$, there exists $m \in D$ such that $S_n qU$, for all $n \geq m$ ($n \in D$).

**THEOREM 2.7.** A fuzzy multifunction $F: X \to Y$ is fuzzy lower δ-precontinuous at a point $x \in X$ iff for every fuzzy point $y_t \leq F(x)$ and for every net $\{x_\alpha : \alpha \in D\}$ in $X$, $\delta_p$-converging to $x$, there exists a subnet $\{z_\beta : \beta \in E\}$ of $\{x_\alpha : \alpha \in D\}$ and there exists a fuzzy point $A^\beta \le F(z_\beta)$, corresponding to each $\beta \in E$ such that the fuzzy net $\{A^\beta : \beta \in E\}$ is fuzzy convergent to $y_t$.

**PROOF.** Let $F$ be fuzzy lower δ-precontinuous at $x \in X$ and $\{x_\alpha : \alpha \in D\}$ be a net in $X$ $\delta_p$-converging to $x$. Also, let $y_t$ be a fuzzy point such that $y_t \le F(x)$. For each fuzzy open q-nbd $V$ of $y_t$, by fuzzy lower δ-precontinuity of $F$, there exists $U_V \in \delta - PO(X)$ containing $x$ such that $F(x)qV$, for all $x \in U_V$. Since $\{x_\alpha : \alpha \in D\}$ is $\delta_p$-convergent to $x$, there exists $\alpha_V \in D$ such that $\beta \ge \alpha_V$ and $\beta \in D \Rightarrow x_\beta \in U_V \Rightarrow F(x_\beta)qV$. Let $D_V = \{\beta \in D : \beta \ge \alpha_V\}$ and put $E = \bigcup_{V \in \Gamma}[D_V \times \{V\}]$ where $\Gamma$ denotes the system of all fuzzy open q-nbds of $y_t$. Clearly $E$ is a directed set under "$\succeq$" given by $(\alpha,V) \succeq (\alpha',V')$ iff $\alpha \ge \alpha'$ in $D$ and $V \le V'$. For any $\beta$ (=($\alpha,V$)) $\in E$, set $z_\beta = x_\alpha$. Then $\{z_\beta : \beta \in E\}$ is a subnet of the net $\{x_\alpha : \alpha \in D\}$. Infact, for any $\alpha \in D$, consider any $(\alpha_V, V) \in E$ and find $\alpha' \in D$ such that $\alpha' \ge \alpha, \alpha_V$. Then $(\alpha', V) \in E$ such that whenever $(\alpha'', W) \in E$ (where $W \in \Gamma$) with $(\alpha'', W) \succeq (\alpha', V)$, one has $\alpha'' \ge \alpha'$. For any $\beta$ (=($\alpha,V$)) $\in E$, we have $F(x_\alpha)qV$ so that $F(z_\beta)qV$. Choose a fuzzy point $A^\beta \le F(z_\beta)$ such that $A^\beta qV$. Let $W \in \Gamma$ be arbitrary. Now $\beta(= (\alpha_W, W)) \in E$ be such that $F(z_\beta)qW$. If $\gamma(= (\alpha, V')) \in E$ with $(\alpha, V') \succeq (\alpha_W, W)$, then $\alpha \ge \alpha_W$ and $V' \le W$. Also $A^\gamma qV' \le W \Rightarrow A^\gamma qW$. Thus $\{A^\beta : \beta \in E\}$ is fuzzy convergent to $y_t$.

Conversely, let $F$ be not fuzzy lower δ-precontinuous at $x$. Then there exists a fuzzy open set $G$ in $Y$ such that $x \in F^-(G)$, and for every $U \in \delta - PO(X)$ containing $x$, there exists $x_U \in U$ for which $F(x_U)\bar{q}G$. Then $\{x_U : U \in \Gamma\}$ where $\Gamma$ is the system of all δ-preopen sets in $X$ containing $x$ (directed by inclusion relation) is a net in $X$, which $\delta_p$-converges to $x$. Let $y_t \le F(x)$ be such that $y_t q\, G$ (such $y_t$ exists as $F(x)q\, G$). By hypothesis, there is a subnet $\{z_W : W \in (\Sigma, \succcurlyeq)\}$ of the net $\{x_U : U \in \Gamma\}$ and corresponding to each $W \in \Sigma$, there exists a fuzzy point $A^W \le F(z_W)$ such that the fuzzy net $\{A^W : W \in\in \Sigma\}$ is fuzzy convergent to $y_t$. Since $G$ is a fuzzy open q-nbd of $y_t$, there exists $W_0' \in \Sigma$ such that $A^W qG$, for all $W \succcurlyeq W_0'$ ($W \in \Sigma$) … (1). Now, since

$\{z_W : W \in \Sigma\}$ is a subnet of $\{x_U : U \in \Gamma\}$, there exists a function $\varphi: \Sigma \to \Gamma$ which is cofinal and $z_W = x_{\varphi(W)}$, for each $W \in \Sigma$. Consider any $U \in \Gamma$. Then there exists $W_0^{"} \in \Sigma$ such that $\varphi(W) \subseteq U$ for each $W \geqslant W_0^{"}$ in $\Sigma$ ($W \in \Sigma$). Let $W \in \Sigma$ be such that $W \geqslant W_0^{'}, W_0^{"}$. Then $W \geqslant W_0^{'}$ and hence $A^W \, q \, G$. Also, $z_W = x_{\varphi(W)}$ and hence $F(z_W)\bar{q}G$. But $A^W \leq F(z_W) \Rightarrow A^W \bar{q} \, G$, contradicting (1).

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